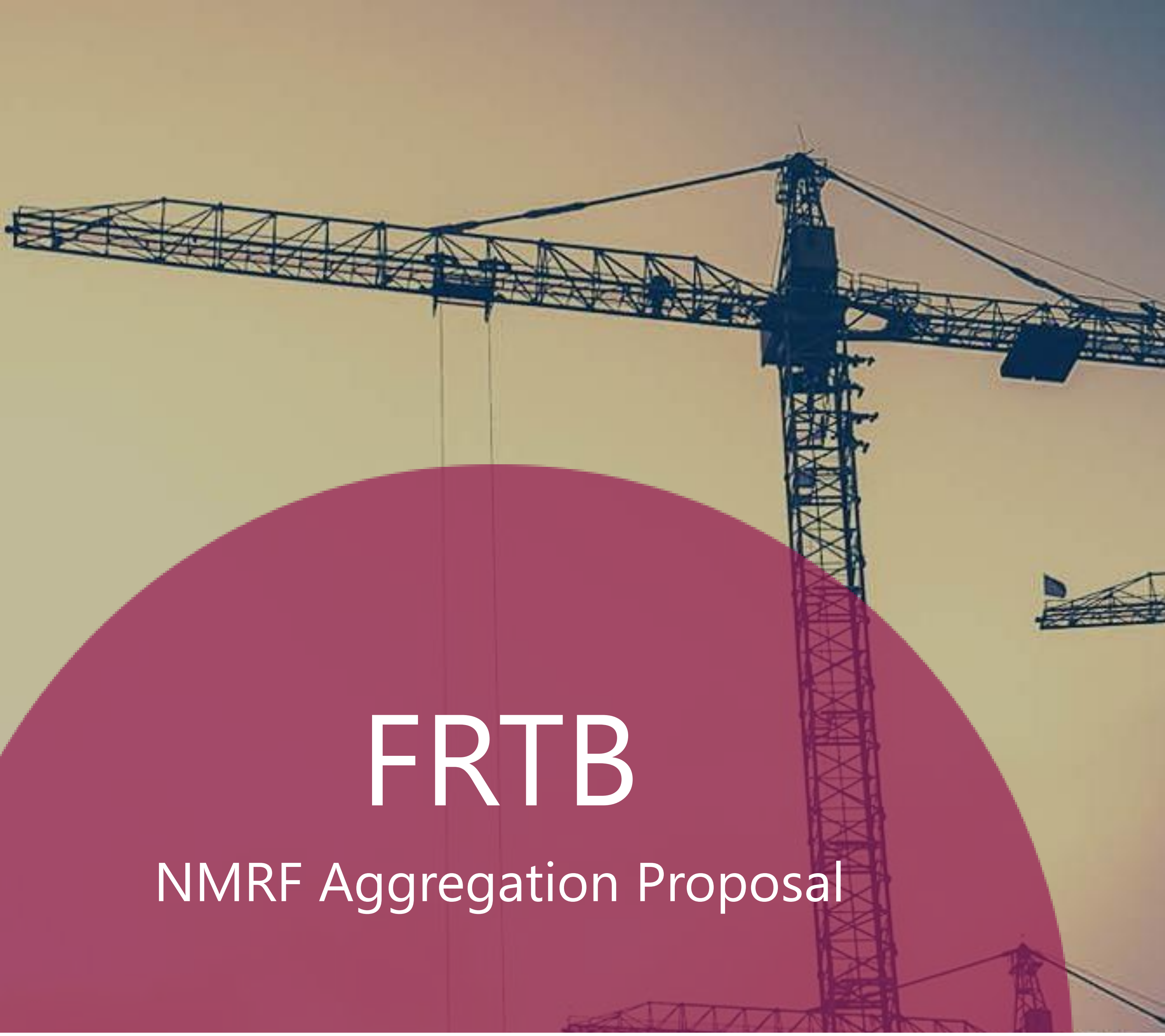

FRTB

NMRF Aggregation Proposal

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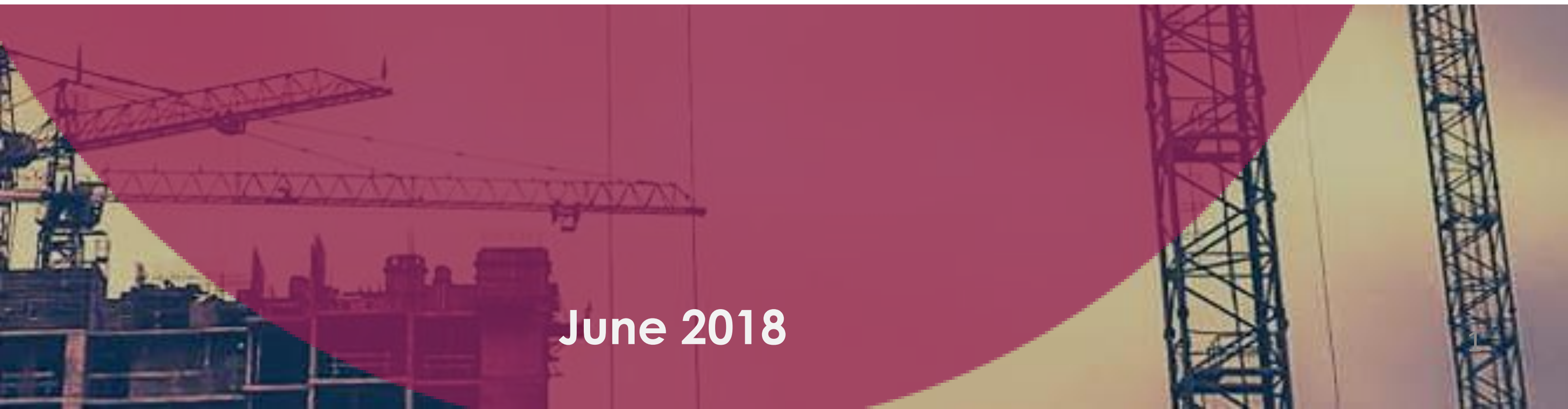
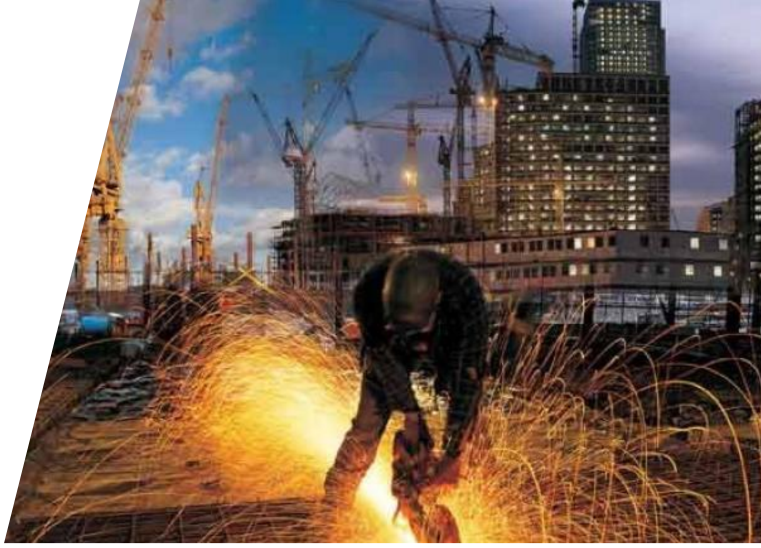


June 2018

Agenda

1. Proposal on NMRF aggregation



Rationale: The current proposal for NMRF aggregation (simple sum) is extremely conservative, to the point that there is no credible correlation structure that could yield such a capital addon.
The proposal below offers a robust method to calculate an addon under the **worst credible correlation**.
We suggest that this is an appropriate method to assess the required capital for NMRFs.

The method which is developed below is transparent, easy to communicate to all stakeholders, and **incentivizes bank to produce high quality and robust models** by relying on measures performed on the model outputs (1) for the estimate of the credible range of correlations.

We first develop why allowing aggregation under a zero-correlation assumption provided that the appropriateness of the assumption is proved might be un-workable (a), why credible correlation ranges are assessable (b), and how a worst credible correlation assumption might be calculated (c).
Language for inclusion in the amended regulation is proposed (d). In conclusion, we indicate the benefits that this method would yield (e).

The proposed method adds a degree of complexity to the NMRF calculation, but:

- the idea is conceptually sound, easy to communicate and provably conservative,
- it doesn't rely on un-provable correlations,
- the added complexity mostly is an analysis of residual (1) correlations which brings risk management benefits,
- this method allows to use the same rule for all sorts of NMRFs, including for credit and equities,
- it allows to materially reduce the capital addon, but in a way which depends on the risk management model quality and on the actual diversification of those NMRFs.

[1] *Most banks will use projections and other transformations in order to reduce NMRFs to a core unobservable component, or "residual". This method penalizes poor transformations which would produce chaotic or correlated Time Series.*

1.1. On the ability to prove correlation assumptions

Correlation assumptions, including the proposed aggregation under an assumption of zero-correlation are typically difficult to prove, even when available Time Series are of high quality for reasons set out below. Many NMRFs will not be associated with high quality historical data.

a) Correlation estimates require many independent observations

When measuring correlation between two Time Series, the error of the estimate is approximately a function of the number of independent returns which can be calculated, that is a function of $\frac{\text{Length of observation}}{\text{Liquidity Horizon}}$ (2).
The error when measuring correlations tend to be large; for instance, measuring the correlation between two uncorrelated Gaussian series of 100 returns, the standard deviation of the measured correlation is **10.0%** assuming a constant volatility.
It is not possible to assess correlations in a stressed period using a Liquidity Horizon that would be long enough to match NMRF Liquidity Horizons. Even using 10 days. Liquidity Horizon, it takes 4 years to get 100 independent points, and volatility is variable in reality.

b) Variable volatility increase the error of correlation measurements.

The noise when measuring the correlation between two Time Series is increased by a non-constant distribution. NMRF stresses will be calibrated on period of stresses, so with an expectation of a different volatility regime in part of the historical data. Even the existence of a few days under a different volatility regime drastically deteriorates the quality of measured correlations.

If, in the example above, out of 100 returns, 10 returns are simulated with a 4-fold increase in volatility, the standard deviation of the measured correlation jumps to **19.8%**. A single day with a 4-fold increase would increase volatility to **15.8%.**

c) It is not generally possible to assess the reason for the variability of correlation measures.

The existence of volatility regime in stressed periods is generally obvious to human observers, but measuring the effect of the change in volatility (which may or may not be gradual) of the estimates of correlations is impossible unless strong assumptions are first made, such as the invariability of correlations.
This assumption is equivalent to assessing individual stresses on the stressed period, but assessing the lack of correlation in a non-stressed period, and the amount of data required for such an analysis might still be more than is available.

In the example above, observing an average correlation of 0, but a large standard deviation doesn't disprove the 0-correlation assumption, but neither proves its appropriateness.
Reaching an average correlation of 0 is straightforward (3), but justifying that in so-doing, residuals can be represented as independent random variables might not be achievable (4).

[2] As shown in Part 3, the use of overlapping returns doesn't improve significantly the variability of measured correlations. In Part 3, the context of DRC is taken, and the use of 1 year return over 11 years (that is 2500 returns) yields an accuracy which is comparable to this when using 14 independent returns, a lot closer to 11 than to 2500.
[3] It is an effect of a Least Square regression, regardless of whether the assumption of a constant volatility is met.
[4] Where different volatility regimes are present, a Least Square regression will give a disproportionate weight to those returns with high volatility, resulting in an inaccurate regression. Reducing the weight on stressed days is akin to assessing correlations mostly on non-stressed periods.

1.2. On the ability to assess correlation ranges

It can be difficult to prove a correlation assumption, in particular when the period of measurement is short and when volatilities are variable.
The difficulty in that assessment is that measured correlations can be extremely noisy in the periods which are the most relevant to a risk model. A consequence of that noise is that the range of measured correlations is wider than the range of underlying correlations (5).
If an aggregation method relies on a range of correlation rather than on a single correlation, and that a wider range of possible correlations yields a more conservative aggregation, then the use of that noisy estimate of correlation ranges is conservative.
It is possible, when a sufficient set of Time Series is available to use a confidence interval measured on historical correlation as a conservative proxy of the confidence interval on the underlying correlation (6). In performing that estimate, only Risk Factors which underwent the same projection/transformations should be considered as a set.

In the context of NMRFs, there will generally be a sufficient number of similar Risk Factors inside each Standard Approach bucket (7) to produce a good quality and conservative estimate of the credible range of correlations both between Risk Factors of that bucket and between Risk Factors of that bucket and other Risk Factors that would be aggregated in a single NMRF capital addon.

[5] Measured correlation = Genuine correlation + Measurement Noise. The amplitude of the noise decreases when the genuine correlation gets closer to -100% or +100%, but this is unlikely to be material when genuine correlation levels are within [-50%, +50%] heteroscedasticity doesn't prevent the use of an empirical confidence interval.
[6] When a large set of high quality data is available, it is possible to refine the interval.
[7] That is, for Internal Model RF, that sensitivities to those Internal Model Risk Factors would accrue in a Standard Approach Risk Factor of that bucket.

1.3. How a calculation could be performed

In order to not recognize correlations which are not recognized in the SBA aggregation framework, we propose a separate aggregation for each Risk Class and Risk Type: Separate NMRF capital addon would be calculated for IR-Delta, IR-Vega, FX-Delta, etc.
Firms would add to the framework relevant Risk Types which don't exist in the Standard Approach, such as IR- Correlation, Equity-Correlation etc.
Those are designated below as Risk Class/Type or RC/T.

The first step of the calculation is an estimate of the ES equivalent stresses.
Those stresses are not converted into capital addons (as the sign of the stresses matter) but in signed charges.
Where the shock "up" result in a P&L of x and the shock "down" in y, the charge associated with Risk Factor i is:

$$C(i)=\mathrm{Sign}(x)\times \mathrm{Max}(|x|,|y|)$$

The second step is an estimate of correlation ranges.
For each bucket $b \in \{1 \ldots N\}$, two correlation ranges are estimated:

$r_b^{Bucket} = [\rho_1, \rho_2]$ such that $[\rho_1, \rho_2]$ is the 90% confidence interval for the correlation between two Risk Factor within bucket b.

$r_b^{RC/T} = [\rho_3, \rho_4]$ such that $[\rho_3, \rho_4]$ is the 90% confidence interval for the correlation between a Risk Factor of b and a Risk Factor of the same RC/T that do not belong to b.

The aggregation is performed under the assumption that each Risk Factor can be modelled as the sum of three Gaussian random variables, one for the Bucket common factor, one for the RC/T common factor, and a noise.

The aggregation is:

$$\mathrm{SES} = \max \sqrt{\left(\sum_{b=1\ldots N}\left(\sum_{i\in b}\rho_b^{RC/T}C(i)\right)\right)^2 + \sum_{b=1\ldots N}\left(\sum_{i\in b}\rho_b^{Bucket}C(i)\right)^2 + \sum_{b=1\ldots N}\left(\sum_{i\in b}\left(1-\left(\rho_b^{RC/T}\right)^2-\left(\rho_b^{Bucket}\right)^2\right)\left(C(i)\right)^2\right)}$$

Where for each bucket b,

$$\rho_b^{Bucket} \in r_b^{Bucket}$$

$$\rho_b^{RC/T} \in r_b^{RC/T} \cap \left[0\%, \sqrt{1-\left(\rho_j^{Bucket}\right)^2}\right]$$

Regulators might want to mandate the correlation ranges until the own assessment of correlation ranges have been subject to internal or external review. In that context, the suggested ranges are:

- $r_b^{Bucket} = [0\%, 100\%]$ for NMRF identical (except for granularity) to a Standard Approach Risk Factor,
- $r_b^{Bucket} = [-50\%, 100\%]$ for NMRF which are residual from the projection of a Standard Approach Risk Factor,
- $r_b^{Bucket} = [-100\%, 100\%]$ for all others,
- $r_b^{RC/T} = [-100\%, 100\%]$

The calculation of the maximum prescribed by the formula above can be produced in a separate system from this used for the production of ES, and is not challenging to implement (8).

The underlying idea is that the Risk Factor i which belongs to bucket b is modelled as:

$$\mathrm{RF_i} = \mathrm{K} \times \mathrm{C(i)} \left[\rho_1 \mathrm{B_b} + \rho_2 \mathrm{RCT} + \sqrt{1 - \rho_1^2 - \rho_2^2}\varepsilon_i\right]$$

Where $\mathrm{B_b}, \mathrm{RCT}, \varepsilon_i \sim \mathcal{N}(0,1)$ and K is the coefficient that converts an ES equivalent into a volatility equivalent.
A volatility for the sum of all Risk Factors in each RC/T can be calculated, maximized and converted back into an ES equivalent (K disappears from the whole calculation).

[8] *A genetic algorithm would be easy to implement and efficient; brute force methods could be used as well.*

1.4. Proposed wording

"Stresses associated to Non Modellable Risk Factors will be aggregated separately for each Risk Class and Risk Type under the worst credible correlation structure. The estimate of the worst credible correlation structure will meet the following:

i. Correlation ranges are assessed with a 90% confidence,
ii. In estimating correlation ranges, only Risk Factors which are sufficiently similar will be used to produce correlation ranges,
iii. The correlation structure allows for at least one explanatory variable per Bucket (as defined in Standard Approach) and one per Risk Type/Class."

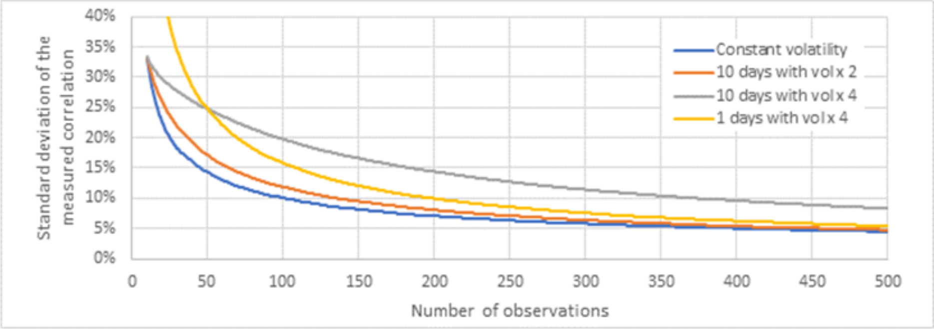
1.5. Benefits of the method

We believe that this method would both allow a prudent calculation of the capital addon for NMRFs and render the charge more risk sensitive.

Further benefits include:

i. That method is clearly conservative: It doesn't assume a correlation structure, but aggregates under a worst possible correlation structure,
ii. The method is adjustable: it is possible to increase or decrease the charge by requiring a higher/lower degree of confidence in the correlation estimates, or by requiring estimates to be calculated with a longer liquidity horizon or on a shorter stressed period (9),
iii. It incentivizes the collection of high quality data, as better data would reduce the range of estimated correlations,
iv. It would strengthen the bank's modelling and increase the scrutiny of the correlation of the least observable Risk Factors.

[9] *A shorter period or a longer horizon both reduce the amount of data available for the calculation of correlations, and directly widens the range of the confidence interval.*
Below a graph of the standard deviation of measured correlations as a function of the number of independent observation points and the existence or not of 10/1 returns with high volatilities.



2. Proposal on modellability assessment

The consultative paper puts forward a bucketing method, which relies on fixed buckets.
It is the industry as well as our opinion that fixed buckets are not the best approach, as those buckets might be more or less granular than is actually required, depending on the Risk Factors and on the bank's trading strategies.
A change in bucketting in major curves would also constitute a material model change, so bucketting could not easily be adapted to market or trading patterns.

While most the industry considers that allowing banks to define their own buckets is a suitable alternative solution, we understand that regulators might be reluctant to allow such a method, as it might not be stable (10), might be difficult to compare across banks, and might be overly reliant on firm's modelling choices.

We therefore suggest a solution which uses a similar rationale but is more stable, easier to compare between banks, and which would give more leeway for regulators to set industry-wide guidance and firm-specific requirements where needed.

Where bucketting assumes that there are discrete and non-overlapping set of Risk Factors that should be assessed together, we suggest to use proxy-observation, where each Risk Factor is allowed to proxy-observe other Risk Factors that are extremely similar to it.
For instance, each Risk Factor may be used to proxy-observe all Risk Factors which have a greater than 98% correlation with it.

Example: Interest Rate curve
Bucketting: When a trade is observed in the]3Y , 5Y] range, this trade counts towards the assessment of modellability of all Risk Factors in that]3Y , 5Y] range.
Proxy-observation: When a trade is observed on the 4.5Y tenor, this trade counts towards the assessment of modellability of all Risk Factors that are sufficiently correlated to that point, for instance 4Y to 6Y. It doesn't observe the 3Y which is too different, but observes the 6Y point, in another bucket, which is sufficiently similar.

Benefits:
- Stability of the method. That stability better allows internal and external auditors to challenge assumptions.
- Comparison between banks is easier (which RF are proxy-observed by which other RF) since there is no boundary effect.

[10] *For firms to use their own buckets, these buckets would need to be assessed from correlation between Risk Factors or other quantitative methods.*
Because all Risk Factors need to belong to a bucket, and some Risk Factors will move in and out of buckets, those buckets could be unstable. For instance if the 2Y tenor drops out of the [0Y , 2Y] bucket, it will have a domino effect and change the definition of all buckets, and possibly the number of buckets.
It will also affect modellability assessment and could have a significant impact on the NMRF charge.

3. Calibration of DRC correlations

Section 186-b requires that "correlations must be based on data covering a period of 10 years that includes a period of stress as defined in paragraph 181(d), and based on a one-year liquidity horizon."

As indicated above (1-a), the use of overlapping returns do no materially improve the accuracy of calculated correlations (11).

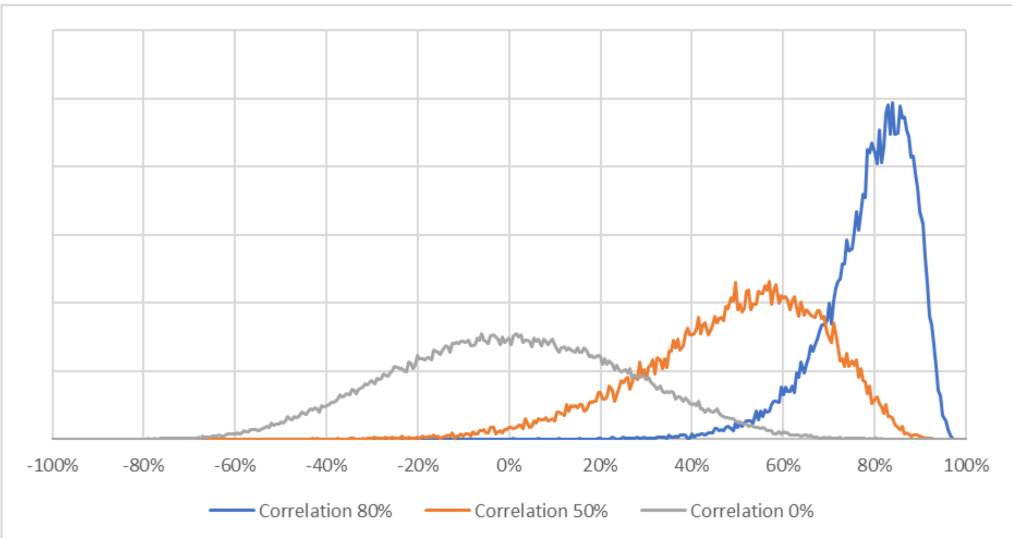
The 95% confidence interval for the measure between two uncorrelated series over 11 years and using 1 year overlapping returns is [-42% , 42%].
The interval for two series with a 50% correlation between one another is [12.5% , 76.5%].
The interval for two series with a 80% correlation between one another is [60% , 92%].

It is a real possibility that correlations be severely mis-estimated on some key pairs of obligors, and that it leads to an erroneous assessment of required capital.

We suggest that the requirement to use a one-year liquidity horizon when calculating correlations be removed.

Firms may consider the effect of using Liquidity Horizon that is too short when calibrating correlations, but merely to eliminate bias (when the average correlations vary depending on liquidity horizon).
Our experience also shows that longer Liquidity Horizon tend to produce lower correlation levels (12) and that most bank's DRC increase with correlation, so a longer Liquidity Horizon tend be less conservative.

Graph: Distribution of measured correlation on two series of 2 749 returns with correlations of 80%/50%/0% when measuring the correlation on 2500 sums of 250 consecutive returns.
This simulates 1Y overlapping returns, with 10Y of return history (so 11Y of total history)



[11] Specifically, using 10 years (plus 1 year of preceding data) of overlapping returns of 1 year each (250 days), the standard deviation of the measured correlation, despite the 2750 points of underlying data is only moderately betted than that when using 11 returns: 25.27% vs 31.36% (measured when simulating uncorrelated series).
[12] After an initial increase which reflects the elimination of noise caused by closing and different closing times.

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