The Adjoint Algorithmic Differentiation Trend

June 2023

Definition

Who would turn down being offered to do more with fewer resources?

Not that long ago, calculating all first order derivatives of a complex pricing function (at the expense of one additional computation cost!) would have sounded like pure fantasy.

Surprisingly, mathematicians had – for a while – in their toolkit a fabulous and elementary technic: the chain rule – which provides Adjoint Differentiation principle, and subsequent formulae. But we've only witnessed its massive adoption (within trading rooms, quant teams, etc.) very recently.

Why this sudden popularity, especially in the FRTB-CVA context? What exactly can it offer to investment banks?

Following interviews with our European clients, we are glad to offer you synthetic insights on the possibilities really offered by AAD, beyond dreams & fantasies.

Adjoint Algorithmic Differentiation (AAD), is a mathematical approach to accelerate the computation of a large number of derivatives of complex mathematical formulas without approximation. It is an alternative to the historical Finite Difference approach built on top of Monte-Carlo engines.

It is a 2-step process:

- It breaks down the formula into a series of elementary operations whose derivatives are easily and quickly computed
- It then reuses those individual derivatives to deduce the derivative of the complex formula

The first step can be achieved via an automated process usually relying on **Machine Learning techniques** (backward graph creation via neural networks) or can be done manually. The later approach is called Adjoint Differentiation (AD).

Sensitivity Calc Reminder

Finite Differences

1. Run base_price (e.g.: diffusion -> payoff evaluation -> discounting-> price)
2. Get shocked_price by running a pricing with application of normalized bump to an underlying risk factor

Sensitivity = shocked_price – base_price

AAD methodology

1. Obtain the backward graph by running a regular base_price
2. Directly computes all the differentials in one run with the same run time, independently of the number of derivatives

How to use of AAD with a simple example

Let's consider the following function:

$$z = f(x_1, x_2, x_3) = e^{x_1} \times 4x_2 \times x_3$$

To determine the forward graph (**step 1**), we break the z function into elementary operations from left to right. Then we reuse these elementary operations to infer the backward graph and deduce the derivatives of the z function (**step 2**)

Step 1 – Forward graph

z has 3 variables: x_1, x_2, x_3

1. The first step is to evaluate exp(x1)
2. We store the result in v_1
3. Then we evaluate $4x_2$ and we store it in v_2
4. We are now able to evaluate the multiplication exp(x1) x $4x_2 = v_1$ x v_2. This is the first part of z
5. The last operation is the multiplication of v_3 with x_3 to get z

Forward Graph

1. x_1, x_2, x_3
2. $v_1 = e^{x_1}$
3. $v_2 = 4x_2$
4. $v_3 = v_1.v_2$
5. $z = v_3.x_3$

Step 2 – Backward graph

1. We start with the step 5 from the Forward Graph, and we derive z with respect to v_3, which will also give the result for x_3.
2. At step 4, we derive with respect to v_1 then v_2. Steps 3 and 2 give the results regarding the derivatives with respect to x_2 and x_1.
3. On this simple example, we notice the AAD gives the same results as a manual derivation (AD) of the z function

Backward Graph: $f(x_1, x_2, x_3) \rightarrow \frac{df}{dz} = 1$

$5.\ z = v_3 \times x_3$

$\rightarrow \frac{df}{dv_3} = \frac{df}{dz}.\frac{dz}{dv_3} = 1.x_3 = x_3$

$\rightarrow \frac{df}{dx_3} = \frac{df}{dz} \times \frac{dz}{dx_3} = 1.v_3 = v_3 = v_1.v_2 = e^{x_1}.4x_2$

$4.\ v_3 = v_1 \times v_2$

$\rightarrow \frac{df}{dv_1} = \frac{df}{dv_3}.\frac{dv_3}{dv_1} = x_3.v_2,$

$\rightarrow \frac{df}{dv_2} = \frac{df}{dv_3}.\frac{dv_3}{dv_2} = x_3.v_1$

$3.\ v_2 = 4x_2$

$\rightarrow \frac{df}{dx_2} = \frac{df}{dv_2}.\frac{dv_2}{dx_2} = x_3.v_1.4 = e^{x_1}.4.x_3$

$2.\ v_1 = e^{x_1}$

$\rightarrow \frac{df}{dx_1} = \frac{df}{dv_1}.\frac{dv_1}{dx_1} = x_3.v_2.e^{x_1} = e^{x_1}.4x_2.x_3$

AAD implementation and maintenance challenge

AAD adoption pros

ADD main benefit is to drastically decrease the computation time of several complex derivatives.

1. Within the XVA context, the performance impact is an opportunity to increase the scope of daily risk sensitivities, delivering more accurate P&L explains and better risk management, while **containing hardware costs.**

2. It helps securing the **delivery SLA of daily risk sensitivities to the XVA desk**.

3. It is a facilitator to **efficiently generate sensitivities** required by the standardized approach of the FRTB CVA capital charge. It is to be noted that the approach is explicitly mentioned as "acceptable" into the BCBS MAR50 framework Q&A.

4. Outside the XVA framework, AAD is often used to **calculate risk sensitivities on complex options** that usually require Monte-Carlo pricing engines.

5. On the mathematical point of view, it is the purest obtainable derivatives, often misunderstood because it is close to sensitivities, often relied upon in the financial industry.

AAD adoption cons

While computing first-order risk derivatives has already been successfully implemented, AAD is **not yet a proven solution for second-order derivatives** (e.g., gammas and x-gamma, which are not in scope of FRTB CVA) upon which some of our clients are putting effort.

Careful thoughts should be given to on-going monitoring indicators and intermediary calculation steps to enable Risk Management trend analysis and regulatory approval.

While the foundation of AAD is nothing but recent, it still is a radical change for most banks and the **transformation costs and frictions can be both significant in intensity as numerous** in source:

- **Project Management** ~ costs, quality & delays, as dead ends can be costly and amount in millions €
- **Framework selection** ~ choosing the right tools (e.g., PyTorch from Meta, C++ overloading with dual numbers)
- **Human skills** ~ having the right people to implement and then maintain it
- **Regulatory approval** ~ having the official theoretical sign-off was only granted very recently by the BCBS
- **Model Monitoring Frame**work ~ needs to be present and robust to ensure performance remains at the right level.

AAD : Algorithmic Adjoint Differentiation

While AD leaves to the developer the task of hard-writing the computation graph for each sub step, AAD manages (via the use of state-of-the-art packages, like PyTorch – or equivalently LibTorch for C++) to generate the computation graph where all operations must be recorded. Once obtained, it is quite easy to navigate it backwards. However, the very nature of financial computations (massive, parallelized & distributed) brings the natural complexities that were responsible for AAD's late adoption in finance.

AD : Adjoint Differentiation

As Antoine Savine (finance's AAD pioneer) summarized it in his renowned publication Differential Machine Learning (2020), "the devil lies in practical details": AD implementation can – technically – be achieved but, at an excessively high cost (each line of code will need a developer review, consequently manual and quite prone to error) doubled with increased maintenance complexity (any update in 'value code' needs be reflected in 'AD code')

Use Case
xVA

A single xVA calculation requires 3 steps:

1. Market data simulation across a high number of Monte-Carlo path (2 000 to 5 000 paths are common) over a long duration (30 years is not unfrequent!)
2. Trade portfolio repricing for each Monte-Carlo path and across default dates (usually around 100 dates) and over exposure dates (ranging around 2000 dates, depending on portfolios, for MPOR considerations)
3. Counterparty-level aggregation based on each Monte-Carlo path and default date valuation, aggregated by time-bucket

A single xVA calculation is one of the largest calculations in an Investment Bank. The order of magnitude is around 6hrs calculations over 10k computation cores.

The SA-CVA methodology in FRTB CVA requires calculating 300 to 500 sensitivities for each bank for their capital requirement calculation. On top of this monthly calculation, SA-CVA approval relies on the trading desk leveraging on similar sensitivities for its risk management. This usually means calculating daily 500 to 2000 sensitivities daily. This could represent a 100-fold increase compared to a single xVA calculation.

Voice of the Customer

"With the help of AAD, we managed to generate 7 millions sensitivities within a single 1h10 run, whereas the Finite Differences method would have brought the same result in an unacceptable 1-week time frame. We are gaining a factor 150 here."

xVA Quantitative Research
Managing Director - French GSIB

Implementation

Two notorious (and public) implementations can be used as a source of inspiration for potential adaptation, and are observed at Danske Bank (home to finance's AAD pioneer Antoine Savine) and Nova Scotia Bank (for which they were granted Risk.Net Award 2021)

Use Case
Exotic Option pricing

Markets & Customers

When markets enter a massive downfall, banks tend to put product offerings on hold, blaming the risk assessment complexity of exotic books

Our client, Natixis

Risk management proved insufficient on autocalls products as the bank suffered heavy losses in 2018, during the Asian market rout

Following change of leadership, Natixis decided to strongly enhance its Risk Management Framework by:

1. **Deploying LSV Model** (Local Stochastic Volatility) powerful analytics across all books & asset-classes
2. **Combining it with AAD** to generate very granular and detailed sensitivities
3. **Running shock scenarios** over the bespoke 2W horizon – designed in JV with the economic research team
4. **Aggregating a central KPI** (Hedge Complexity Score based on exotic risks of convexity, concentration, correlation) at various level

Resulting in better Risk Management steering and Product Offering oversight by an enhanced view on tail risk

2022 equity market volatility offered a successful baptism-by-fire.
In-spite of an altered market landscape 1Y revenue target was hit within 6M !

“Instead of preparing for the last crisis, Natixis has given itself the best possible chance of predicting the next one.

Risk.net, 26 Jan 2023”

Voice of the Customer

“*This improved risk framework enabled us not only to better manage our risks, but to do more business*

Michael Haize
Natixis CIB Global Head of Global Markets”

How Capteo can assist you

Conclusion

The AAD methodology offers many benefits of high value for financial institutions: **performance, speed, accuracy**. While banks have always been economically interested in such extra-value (to optimize either their PnL or Risk Management framework), **the regulatory context and the capital requirements are now making it non-optional.**

AAD's implementation ROI could be enhanced when **combined with other quantitative technics** as achieved in Differential Machine Learning (Brian Norsk, 2020) where AAD's phenomenal computing potential is sublimated by deep neural networks, where complex sensitivities can be estimated on hypothetical on shocked portfolios.

We believe the most efficient way to lower AAD's implementation costs, **by design a one-shot-process** for any institution, is our very experience, gained through AAD deployment alongside the clients we support.

From our perspective, AAD will offer many applications **combined with innovative AI technics**, thanks to on-the-shelf solutions/packages now available with their rising popularity.

Capteo can support you throughout all steps of AAD related initiatives: FRTB CVA sensitivity computations, or complex pricing performance enhancement…

Especially, we can help you:

- **Define and assess new Business Cases**, where AAD may bring critical value
- **Frame and lead the setup of AAD related projects** with respect to your technological ecosystem and corporate organization
- **Steer and oversee various risks sources**: collective awareness & technicity around AAD, internal financial risks, regulatory, choice of the right technological framework
- **Implement metrics and methodologies** to monitor AAD performances
- **Assist organizations on the on-going use and perennity of AAD** by ensuring a posteriori its flexibility and capacity to deliver value on future challenges

As a strategy and management consulting firm, we specialize in the financial institutions' transformation & evolution, and provide quantitative expertise on complex subjects arising from Front-Office, Risk or Regulatory challenges such as Capital Consumption optimization, FRTB SA-CVA implementation or complex Risk Management framework enhancement.

Contacts

Gabriel LETHU
Senior Partner – Risk & Finance
glethu@capteo.com
Tel : +33 677 566 992

Alexis BUNEL
Traded Risk SME
abunel@capteo.com
Tel : +33 766 891 157

Rony BARMAT
Senior Manager
Front & Counterparty Risk Quant
rbarmat@capteo.com
Tel : +33 628 667 907